but now that shattering of nuclei is possible, and now that rapid means of detection are feasible, there is something to look for. The formation of strange substances and unusual combinations may be expected, and the composite nature even of the proton may yet be demonstrated by the emission of something fractional of extreme instability. Does not the atomic bombardment of aluminium already yield particles of extra long range ?

I make no apology for this surmise. Speculation as a temporary working hypothesis is sometimes suggestive of further experiment, and that is its sole justification. If the tendency of the discussion is to uphold the greater simplicity of the extra-small and extra-massive indivisible positive particle, well and good ; but that would rather close the door on one line of experiment, and it is not well to abandon the mirror-image idea prematurely. The proton *may* be an indivisible ultimate unit ; but that seems unlikely, and we have learnt not to negative the possibility of ascertainable structure lightly. It seems barely credible, now, that it was as an indivisible ultimate unit that we used to regard the atom !

The hypothesis that a proton is built up of positive and negative but otherwise identical electrons may yield a hydrogen nucleus too bulky for the facts, and may otherwise have to be rejected, but the idea at least leaves the door open to the extraordinarily brilliant experimental physicists of to-day, and hence as long as possible may be tentatively and provisionally encouraged.

OLIVER LODGE.

Normanton, Lake, Salisbury.

The Measurement of Intervals.

I CANNOT resist Mr. Cunningham's invitation in his review of my Romanes Lecture (NATURE, Oct. 28, p. 568) to justify more precisely the transition from the picture of world-history as a tangle of world-lines to the scheme of intervals filling a continuum of space-time and demanding non-Euclidean geometry. "Prof. Eddington seems to contemplate as 'measurable' the intervals between pairs of points in this continuum which do not correspond to events in the history of any particle or electron in the material universe. But we wish to ask him how these intervals are in practice to be measured." Mr. Cunningham's point is that the picture which we have to dissect is the *actual* history of the world, and we are not allowed to alter it—to introduce measurements which never were made, or to introduce physically recognisable events at points where nothing actually happened. I accept this limitation. He admits, however, that all measurements that have ever been made are contained in the picture, and, I might add, all measurements that ever will be made. Thus we have a large number of measured intervals available for discussion ; and I think that Mr. Cunningham, like myself, is convinced that the geometry which these measured intervals obey is not exactly Euclidean but is given correctly by Einstein. When once this geometry is determined we proceed to fill all space-time with *calculated* points and intervals ; just as we ordinarily fill all space with calculated points and distances after first determining the geometry by means of a few distances actually measured and a few points actually perceptible. Only a small number of the calculated points and intervals correspond to events and measurements in the historical picture ; but whenever there is a measured value it will agree with the calculated value.

As regards the status in physics of this scheme of calculated points and intervals, it does not seem necessary to make any hypothesis ; indeed, I scarcely know what hypothesis could be made about it. At the back of my mind I vaguely suppose that it is "closely descriptive" of an underlying relation-structure of the actual world ; but whatever that means (if it means anything) it is too indefinite to use as an hypothesis. It is sufficient that we find it profitable to talk about this scheme. But at least its status is in no way inferior to the picture of tangled world-lines which Mr. Cunningham finds it convenient to start from. Material particles and events outside us are not directly observed : they are inferred from the fields (inertial and electromagnetic) which affect our bodies. But the field itself is not directly observed ; it produces disturbances in the bundle of world-lines called a man. Inside the man the disturbance passes from field to matter and matter to field in endless cycle. Who shall say at what phase of the cycle it takes the final plunge into the realm of consciousness and actuality ? Rightly or wrongly the method of science has always been to generalise from observation—to talk about a world which includes all that has been observed and a great deal which has not been observed. The astronomer does not make the *hypothesis* that the moon exists when nobody is observing it ; but he finds it profitable to talk about a conceptual picture which contains a continuously existing moon. The scheme of calculated points and intervals (æther, or field) or of tangled world-lines (matter), or preferably both together, forms the world which the physicist finds it profitable to discuss ; he can scarcely attribute more virtue than that to any world without wandering into metaphysics.

I must dissent entirely from Mr. Cunningham's statement that "any geometrical system whatever may be used for the purpose of attaching intervals." Clearly if a wrong geometrical system is used, the *measured* intervals will expose it by their disagreement. But Mr. Cunningham in this passage seems to use the word interval as though it had no fixed meaning and he could make it mean what he liked. If I recollect rightly, I originally introduced the name "interval," preferring it to the name "line-element" then current, which seemed unsuitable for a physical quantity as savouring too much of pure mathematics. I intended "interval" to mean a definite physical quantity—quite as definite as "energy," for example ; and I desire to guard its meaning jealously. If the meaning of "energy" can be altered at pleasure, it is easy to upset the law of conservation of energy ; and similarly by treating "interval" and "length" as words meaning nothing in particular, Mr. Cunningham has no difficulty in disposing of my contention that the world is not a Euclidean or flat world.

It will be seen that Mr. Cunningham and I are essentially in agreement that the merit of the Einstein scheme of intervals is its simplicity—"profitable to talk about"—rather than some kind of metaphysical significance. He regards it as selected from many other possible schemes because it gives a simple representation of the motion of particles and light-rays. That is a quite good enough reason for selecting it, but it must be borne in mind that it is not the historical reason for choosing it. The fact that it describes the exact motion of Mercury in a particularly simple way was only discovered after the whole scheme had been completed. The interest of Einstein's scheme is that there is, not one reason, but several reasons for selecting it. Not the least important of these reasons is that the scheme expresses the geometry of the world—in the sense in which the

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word "geometry" is commonly understood, *e.g.* by the Board of Education.

One remark as to Newton and the apple, which I intended to typify a supported observer and a continuously falling observer, respectively. If, with Mr. Cunningham, we take the apple to typify an observer at first supported and afterwards free, the apple's view of things is appallingly complicated—compared even with Newton's. But that only the more emphasises the point that the natural simplicity of things may be distorted *ad libitum* by the process of fitting into an unsuitable space-time frame.

A. S. Eddington.

Observatory, Cambridge,
November 3.

I am obliged to the Editor for giving me an opportunity to add a few words in comment upon Prof. Eddington's letter, and I do so in no captious spirit, but because it seems to me that in these very fundamental discussions it is of the utmost importance to clear away as many misunderstandings and difficulties as possible; to recognise that some divergences are merely consequences of viewing the same matter from different points of view, but that others may be due to looseness of thought on one side or the other; and I am glad to be able to recognise that most of the divergence of Prof. Eddington's exposition of the meaning of Einstein's theory from my own understanding of it is merely part of the difference between our natural ways of thinking. But two sentences in Prof. Eddington's letter do sum up my difficulty in regard to his exposition so clearly that I would like to direct attention to them.

"He admits, however, that all measurements that have ever been made are contained in the picture, and, I might add, all measurements that ever will be made. Thus we have a large number of measured intervals available for discussion."

In this sentence Prof. Eddington begs the whole question with which I ventured to end my review of his lecture. All measurements of length and all measurements of time that were ever made are, I agree, in the picture. But who ever measured this physical "interval"? What is the absolute scale of interval, and how is it applied? Again in Prof. Eddington's letter we read: "Clearly if a wrong geometrical system is used, the *measured* intervals will expose it by their disagreement." Unfortunately this is not at all clear to me, and I will try to explain why. So far as I can see, all actual physical measurements are records of observations of coincidences, *e.g.* of marks on a scale with marks on another body. That is to say, they correspond to intersections and concurrences of world lines of distinct physical elements. The significant feature of the four-dimensional picture of the universe is therefore merely the order of arrangement of such concurrences along the world lines of these physical entities. All else is of the nature of an arbitrarily adopted method of description of these orders of arrangement and is not contained in the picture itself. A geometrical system is an analytical means of describing the picture. The concurrences remain and their order is unaltered, no matter how we change our geometrical system. If I adopt a geometrical system other than that of Einstein, I may find the mathematics more complicated, but the actual observable facts recorded are the same—just as the fact of the meeting of the Great Northern, Great Eastern, Midland, and London and North-Western Railways in Cambridge station is quite independent of any particular brand of map or time-table. Of course a map which denied this fact would be wrong—but the adoption of a different geometrical system of attaching what I must not call "interval" to the separateness of two events does not break up a concurrence. It is just because *actual measurements* will not be altered by any change of the geometrical system that I cannot agree with the sentence I have quoted.

E. Cunningham.

St. John's College, Cambridge,
November 11.

The Time-Triangle and Time-Triad in Special Relativity.

Dr. Robb directs attention in Nature of October 28, p. 572, to the fact that there is much confusion of thought with regard to the stationary value of the integral $\int d\sigma$ in the special theory of relativity. When the path is purely temporal, as Dr. Robb was the first to point out, the integral is an absolute maximum, not a minimum. Prof. Eddington has also directed attention to this truth. The following view may be of interest. I give mainly the results, as the precise mathematical proof would occupy too much both of space and time.

Let A, B, C be the vertices (point-instants) of a *pure time-triangle* in the field of special relativity. Suppose C precedes A, and A precedes B *in proper time*; then it may be proved that C precedes B, *i.e.* proper time order is *transitive*. Then if *cosh* C denotes the unit-scalar product of the vectors CA, CB, and if α, β, γ denote the real and positive intervals BC, CA, AB, we have

$$\cosh C = \frac{\alpha^2+\beta^2-\gamma^2}{2\alpha\beta}.$$

It may be proved that the expression on the right-hand side is always positive and is greater than unity. Thus C may be regarded as the real invariant "*hyperbolic angle*" between the temporal vectors CA and CB. This angle has the same metrical value for all observers moving with uniform mutual relative velocities.

It can also be proved that $\alpha > \beta$. Hence, since $\cosh C > 1$,

$$\alpha > \beta + \gamma.$$

That is, *the greatest side of a pure time-triangle is greater than the sum of the other two sides.*

It follows at once that the stationary value of the integral $\int d\sigma$, where the path is purely temporal, is an absolute maximum.

There is thus a real hyperbolic angle between any two co-directional temporal vectors. The triangle ABC has two real "internal" hyperbolic angles (B and C), and one real "external" hyperbolic angle A′. Besides the above formula we have

$$\cosh A' = \frac{\alpha^2-\beta^2-\gamma^2}{2\beta\gamma}, \quad \cosh B = \frac{\gamma^2+\alpha^2-\beta^2}{2\gamma\alpha}.$$

Taking positive signs for intervals and angles, we have

$$\frac{\sinh A'}{\alpha} = \frac{\sinh B}{\beta} = \frac{\sinh C}{\gamma}$$

and $\cosh(B+C) = \cosh A'$.

Thus *the one real external angle of a time-triangle is equal to the sum of the two real internal angles.*

The hyperbolic angle between two co-directional temporal vectors has a perfectly definite physical meaning, if the physics of special relativity is sound. Let CA and CB be the time-axes used by two

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